Occurrence and Behavior of Current Spokes in MPD Arcs

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A pulsed MPD arc has been operated to experimentally determine the occurrence of the rotating spoke mode of operation and to investigate voltage characteristics and spoke frequency of the MPD arc. It was found that the single rotating current spoke occurs at intermediate current levels above a critical magnetic field strength. A comparison of the spoke frequency measurements with the actuator-disk theory of Fay and Cochran shows that the actuator-disk model gives a satisfactory prediction of the behavior of the spoke frequency until a limiting frequency is reached after which the spoke approaches a constant rotational speed which is the same for all gases. The variation in the electrode voltage was found to obey the correlation of Patrick and Schneiderman when the arc was operated in the diffuse mode at low-power levels. In the spoke mode voltages several times this prediction were measured and can be explained in terms of the EMF generated by the rotating spoke. Several criteria are suggested which predict the observed spoke/no-spoke boundaries.

Nomenclature

a_2 = sound speed of gas at arc exit
A = channel cross-sectional area downstream of arc
B = magnetic field strength
C_D = nondimensional drag coefficient
d = radial electrode gap
D = width of arc
e = charge of an electron
E = electric field
I = total current
j = current density
l = mean electrode radius
$\dot{m}$ = mass flow rate
m = mass of a molecule
n_e = electron number density
$\dot{n}$ = molecule number flow rate
T = gas temperature
U = gas speed
U_c = critical speed
V_e = electrode voltage
V_0 = sum of voltage drops in electrode boundary layers
β = constant of order one
ϵ_D = dissociation energy
ϵ_i = ionization energy
μ_0 = permeability of free space
ρ = density
σ = electrical conductivity
ω = angular velocity
$\omega_e\tau_e$ = Hall parameter

Subscripts

r, z, Θ = radial, axial, and azimuthal components

I. Introduction

To measure the magnetic field, current, and electric field distributions in the interelectrode region of the MPD arc, Ekdahl et al.[1] performed experiments with a pulsed quasi-steady MPD arc. It was discovered that the flow in the MPD arc was not steady and axisymmetric but that the current was concentrated in a radial arc or spoke which rotated in the $\mathbf{j} \times \mathbf{B}$ direction. Measurements indicated that the rotating arc formed concentrated anode and cathode spots and that its azimuthal extent was less than 90°. The arc contained no azimuthal (Hall) component of current and the number density within the rotating spoke was an order of magnitude higher than that in the surrounding region. Subsequent experiments in steady flow MPD arcs performed by Larson,[2] Connolly,[3] and Malliaris[4] showed that such rotating current spokes existed in conventional MPD arcs for a wide range of operating conditions. All experiments showed that the current pattern rotated in the amperian direction with a steady frequency on the order of 100 kHz which increased with current I and the applied magnetic field B but decreased with increased propellant mass flow rate $\dot{m}$.

The discovery of the rotating spoke mode of operation raises questions concerning the validity of the interpretation of previous MPD arc experiments[5–8] and those attempts to theoretically explain the operation of the MPD arc which first assume an axisymmetric discharge.[9–11] There is, however, experimental evidence that the MPD arc can operate in both a spoke and a diffuse mode for different values of I, B, and $\dot{m}$.[12,13] In addition, several studies[14–17] have been performed which attempt to describe the physical character of the spoke mode of operation and to predict the conditions under which it will occur. There is, however, insufficient data concerning the values of the three independent variables I, B, and $\dot{m}$ for which the spoke mode occurs for various propellants. In addition, although this is not studied here, no systematic study of the effect of electrode geometry on the occurrence of the spoke mode has been made. Also, there exists disagreement in the literature[4,17] as to the variation of the frequency of the spoke with I, B, and $\dot{m}$; and over-all characteristics such as the electrode voltage for the MPD arc operating in the spoke mode have often not been reported.

In the experiments discussed below spoke occurrence, electrode voltage, and spoke rotational speed were measured in a single quasi-steady MPD arc for a broad range of the three independent parameters I, B, and $\dot{m}$. Propellants investigated were hydrogen, helium, xenon, and argon and were chosen because of their wide range of molecular weight and ionization energy. As explained by Ekdahl et al.,[1] steady MPD arc conditions can be obtained with a pulsed device of 500 μsec duration with the exception of the cathode temperature which does not attain its steady-state value in the experimental test time. (The cathode of the MPD arc used for the present experiments was not heated, but this is not felt to be a significant effect.) The use of pulsed operation eliminated the need of expensive pumping systems and electrode cooling techniques and allowed a larger variation of I (100–5000

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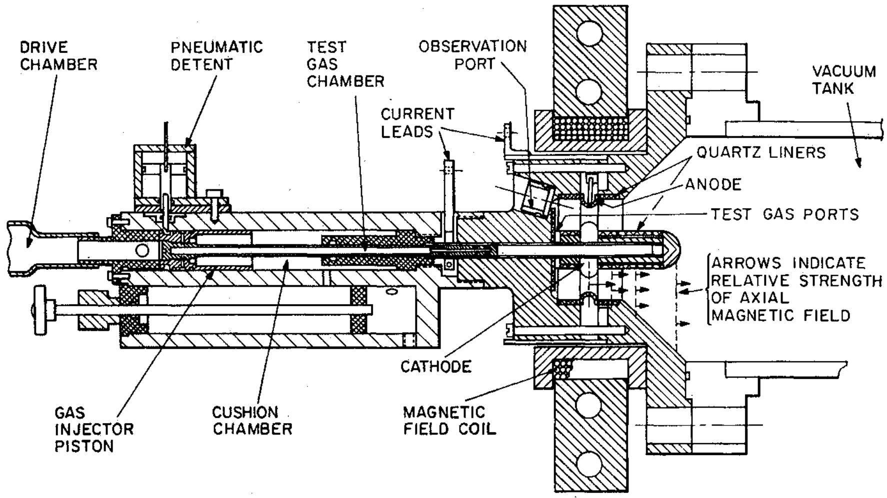

Fig. 1 Cross-sectional view of gas injector and MPD arc's electrode configuration.

amp), B (0.1–2.0 tesla), and $\dot{m}$ (0.056–4.0 g/sec) than could be accomplished in a single conventional steady flow MPD arc.

The analysis of the data is presented in Sec. III of this paper where reference is made primarily to the actuator-disk theory of Ref. 17 which motivated the experiment. However, whenever possible other correlations are suggested which are important from an engineering point of view to predict the occurrence of the spoke mode and the frequency and voltage characteristics of the MPD arc. By designing the electrodes such that they consisted of two concentric copper rings, an attempt was made to constrain the arc for better observation and to more closely approximate the idealized geometries considered in the theoretical treatments of Refs. 15–17. This is discussed in more detail in the next section.

II. Experimental Apparatus and Methods of Measurement

A schematic diagram of the MPD arc showing the electrode configuration, exhaust nozzle, magnetic field coil, and gas injector valve is shown in Fig. 1. The copper cathode, which was 1-cm thick, had an outside diameter of 4 cm and was separated from the concentric copper anode ring by a 1-cm gap. The exhaust nozzle and housing of the MPD arc was constructed from linen base phenolic material and all surfaces adjacent to the electrodes were covered by quartz liners to prevent ablation by the discharge. The gas injector valve is similar to one designed by Avco Everett Research Laboratory[18] and opens in 50 μsec providing an approximately steady flow for 1 msec.

Current was supplied to the electrodes by a lumped parameter transmission-line capacitor bank with six sections, each containing six 100 μf capacitors in parallel and separated by a 5 μh inductor. This circuit provided an approximately square current pulse of 500 μsec duration with currents as high as 10,000 amp. The axial magnetic field was provided by a coil located directly over the electrodes, as is shown in Fig. 1. The current to energize the coil was supplied by a 4800 μf capacitor bank. The axial field strength was nearly uniform over the electrode gap, and the maximum axial field which could be provided was 2.1 tesla. The exhaust nozzle was connected to a 6-in.-diam, 20-ft-long shock tube by an intervening 3-ft-long, 6-in.-diam glass pipe. This long exhaust chamber allowed the experiment to be completed prior to the return of pressure waves reflected from the end wall of the chamber. For all experiments the initial exhaust pressures were 10^{-4} torr or less.

For each run the following techniques were used to investigate the uniformity of the discharge.

1) Frequency pickup photo-optical system. A photo-optical system was used to monitor the light intensity produced by a highly localized region between the electrodes. A $\frac{1}{8}$-in.-diam optical fiber transmitted the light from a lens system to an RCA 6342 A photomultiplier tube which has 10 stages and a maximum sensitivity between 4000 and 5000 Å.

2) $\dot{B}_\theta$ coil. The locally fluctuating azimuthal magnetic field in the electrode region was monitored by a small magnetic field coil. The coil was double-wound and center-grounded,[19] enclosed in a 3 mm o. d. quartz tube, and positioned 1 cm upstream of the electrode gap.

3) Floating probe. To measure the temporal variation of the potential of the gas between the electrodes, a single electrode floating probe was placed between the anode and the cathode. The probe consisted of a 0.025-in.-diam tungsten wire which was inserted from the rear through the observation port (see Fig. 1). The potential difference between this probe and the anode was measured.

III. Experimental Data and Discussion

Existence of Spokes

It became apparent at the onset of the experiments that the character of the discharge depended on the values of the three independent variables I, B, and $\dot{m}$. To illustrate this, Fig. 2 shows the response of the photo-optial system and floating probe for a series of runs in argon at a constant mass flow rate and a fixed axial magnetic field strength. Considering the output of the photomultiplier tube, it is seen that at low currents the variation in the voltage output is small but that as the current is increased there occurs a coherent large fluctuation. (The sharp decrease in the voltage corresponds to an increase in the intensity of light emitted by the plasma.) This fluctuation again reduces in size at high currents and loses its coherent character. Oscillations having a similar behavior appeared in the outputs of the $\dot{B}_\theta$ coil and floating probe. By using two photo-optical systems placed either 90° or 180° apart on the azimuth, it became evident that these fluctuations are caused by a single arc column or spoke rotating in the $\mathbf{j} \times \mathbf{B}$ direction.

The quantity of current associated with current fluctuations in the discharge could be estimated by integrating the $\dot{B}_\theta$ coil output and assuming that the current in the fluctuation was concentrated in a single current filament between the electrodes. Using this technique, Fig. 3 shows the percentage of the total current associated with typical fluctuations for runs at a constant argon mass flow rate at different magnetic fields and arc currents. Runs for which a single rotating arc column occurred are designated by the shaded symbols while the other symbols correspond to runs where no distinct fluctuation corresponding to a single rotating spoke occurred. It is seen that rotating spokes containing a large percentage of the total arc current occur at intermediate current levels (600–3000 amp) and at magnetic field strengths above 0.5 tesla.

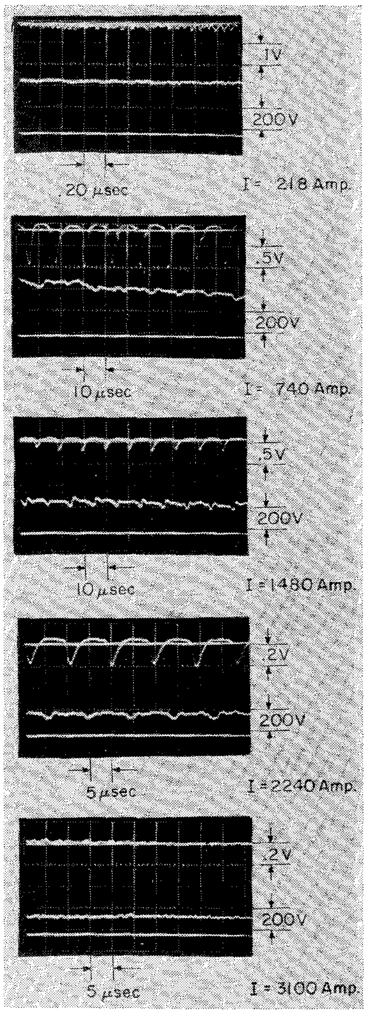

Fig. 2 Oscillograms showing output of photo-optical system (upper traces) and floating probe (lower traces) for a series of runs at an argon mass flow rate of 0.65 g/sec and a magnetic field of 1.0 tesla.

Figure 3 also shows that fluctuations containing relatively large values of current (approaching 20% in Fig. 3 and larger for other conditions) can occur as well as single rotating spokes containing small amounts of current (less than 20%). Because of this, a somewhat arbitrary definition of the spoke mode and the no-spoke mode of operation is made. The discharge is said to be spoked when there occurs a single rotating current arc evidenced by a definite coherent frequency containing 20% or more of the total current. All other runs correspond to the no-spoke mode which is further categorized as being either diffuse or marginal. A diffuse run is one where there are no current fluctuations greater than 20% of the total current, and marginal runs are runs for which random fluctuations having greater than 20% of the total current are observed.

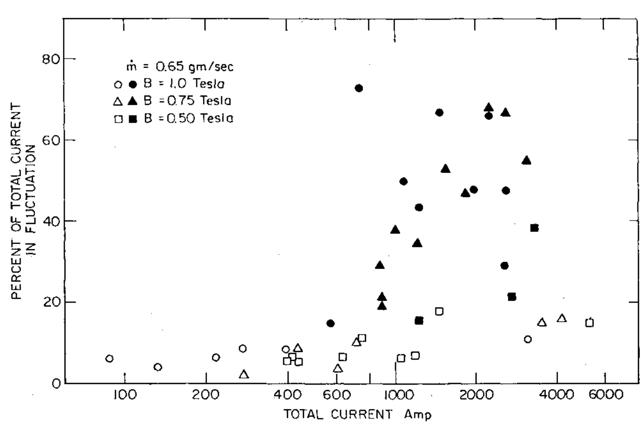

Fig. 3 Percentage of total current in fluctuations the total current delivered to the arc for various magnetic field strengths at an argon mass flow rate of 0.65 g/sec. The shaded points correspond to runs where single rotating spokes occurred.

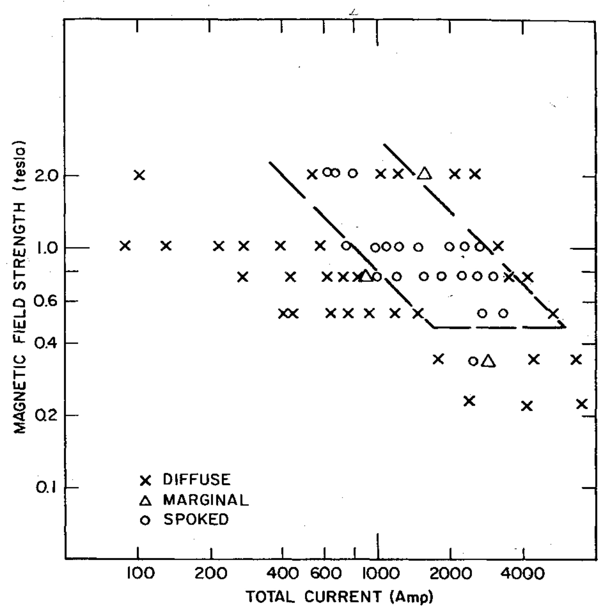

Fig. 4 Chart of the occurrence of rotating current spokes for an argon mass flow rate of 0.65 g/sec.

Using the aforementioned definition, Fig. 4 is a chart made at a constant argon mass flow rate which shows the values of the total arc current and axial magnetic field strength for which the spoke mode occurs. Figure 4 shows that the spoke mode occurs above a critical magnetic field strength and at intermediate current levels. Above the critical magnetic field strength the two lines which separate the spoke and no-spoke region are approximately designated by the equation IB = const. This qualitative behavior is also observed for experiments made using xenon, hydrogen, and helium as propellants. Table 1 gives the critical values for the lines dividing the spoke and no-spoke regions for the mass flow rates which were extensively investigated for argon, xenon, and hydrogen. The criterion for the occurrence of rotating spokes will be discussed later in this section.

Spoke Frequency Measurements

An attempt has been made to find a set of nondimensional parameters which plotted one against the other provide a satisfactory correlation of the spoke frequency data over a significant range of operation. Two nondimensional variables are suggested by the actuator-disk model.[17] This model is based on the physical assumption that the rotating spoke in

Table 1 Critical values of the lines dividing the spoke and no-spoke region

Gas	$\dot{m}$, g/sec	IB',[a] amp tesla	IB''[a] amp tesla	B_{cr}, tesla	IB'_{cal},[b] amp tesla
Argon	0.65	800	2800	0.46	641
Argon	0.35	350	2750	0.20	186
Argon	0.056	320	2200	0.14	4.5
Xenon	0.65	400	5400	0.50	225
Hydrogen	0.0164	400	2000	0.25	100

[a] IB' and IB'' are the constants which characterize the high current and low current boundaries, respectively, as shown in Fig. 4.
[b] Calculated using Eq. (10).

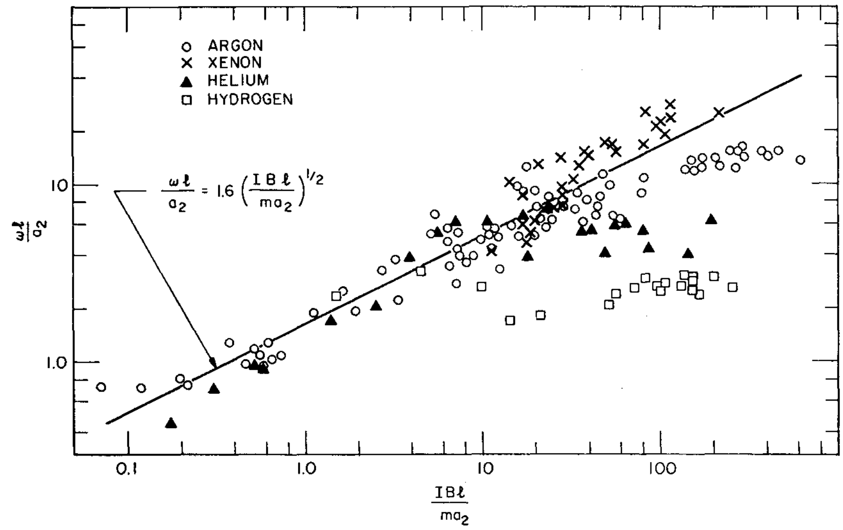

Fig. 5 Correlation of measured spoke frequency as suggested by the actuator-disk theory[17] for runs made with argon, xenon, helium, and hydrogen.

the MPD arc is similar in structure to the magnetically balanced arc column.[20–24] As suggested by magnetically balanced arc experiments,[20–22] the relative velocity W between the flow and a balanced arc column can be related to the current through the arc and the transverse magnetic field by

$$C_D D \rho \, W^2/2 = IB \tag{1}$$

where D is the visible width of the arc column, ρ is the mean density of the impinging flow, and C_D is a drag coefficient which has been found experimentally to be of order one. Using Eq. (1) to predict the azimuthal speed of the rotating arc column in the MPD engine,

$$\omega l = (2IB/\rho D)^{1/2} \tag{2}$$

where ω is the angular speed of the arc column, l is the mean gap radius of the MPD arc, and C_D is assumed to be one. For most MPD arc experiments the actual mass density in the arc region is not available, and therefore it is necessary to make an additional assumption. Using the assumption suggested by Workman[11] that the flow is choked in the exit of the arc region, it is proposed that

$$\rho \approx \dot{m}/a_2 A \tag{3}$$

where $\dot{m}$ is the total mass flow rate, A is the exit channel area, and a_2 is the sound speed of the hot gas emitted from the arc region. Combining Eqs. (2) and (3),

$$\omega l/a_2 = \beta(IBl/\dot{m}a_2)^{1/2} \tag{4}$$

where the arc width is assumed to be of the order of the electrode gap and β is a constant of order one.

In Eq. (4) $\omega l/a_2$ is the spoke Mach number while $IBl/\dot{m}a_2$ is approximately equal to the Mach number based on the calculated azimuthal gas speed at the exit of the discharge. Aside from the direct derivation of the nondimensional quantities from the actuator-disk model, their importance can be seen from analogy with ordinary compressible flows. In Fig. 5 $\omega l/a_2$ is plotted as a function of $IBl/\dot{m}a_2$ for all the spoke frequency measurements made using argon, xenon, helium, and hydrogen as propellants; Eq. (4) is also included with $\beta = 1.6$. The sound speed is calculated to correspond to that of the fully dissociated parent gas at a temperature of 12,000°K. The propellant which was most extensively investigated is argon. The argon data is satisfactorily correlated over the entire range, but agreement with Eq. (4) is observed only in the low-speed region below a critical value of $IBl/\dot{m}a_2 = 30$. For the high-speed region above $IBl/\dot{m}a_2 = 30$ the rotational speed increases at a rate less than that predicted by Eq. (4) becoming almost constant for $IBl/\dot{m}a_2 > 100$. For the data taken with the three other propellants these two characteristic regions were also observed, although for xenon and hydrogen, data was taken only below and above, respectively, the critical value of $IBl/\dot{m}a_2$ which separates the high- and low-speed regions. The two regions are referred to as the high- and low-speed regions because the absolute spoke speed ωl at which the departure from the square root behavior occurs is approximately the same for argon, helium, and hydrogen.

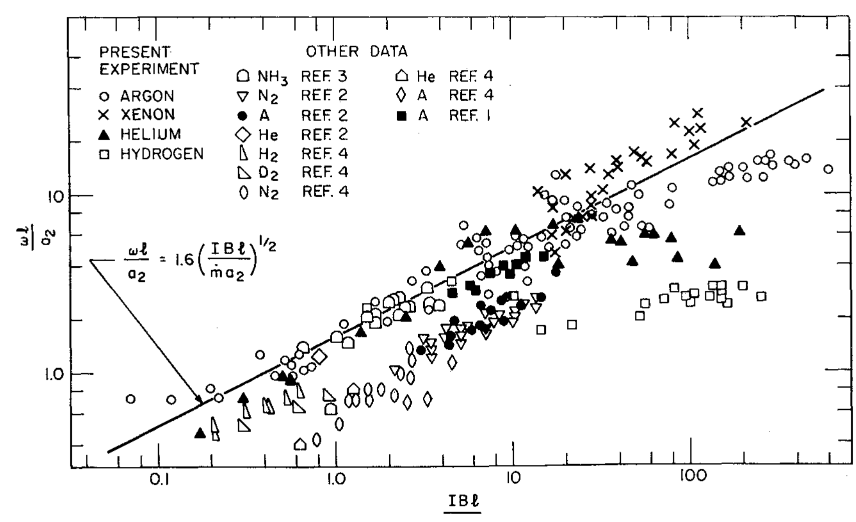

Fig. 6 Comparison of other reported spoke frequency measurements with the present data and the actuator disk theory.

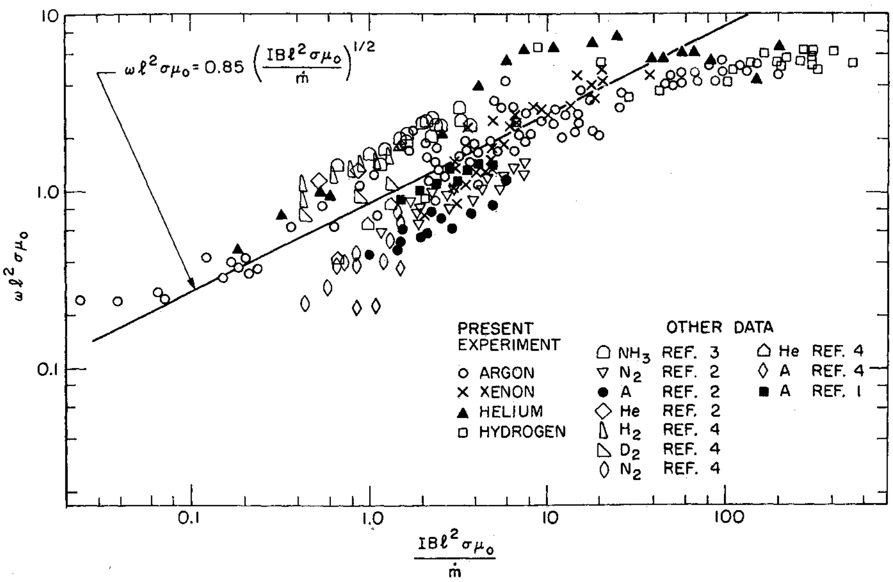

Fig. 7 Comparison of present data and other spoke frequency measurements with correlation suggested for $\omega l^2 \sigma\mu_0 > 1$.

Most conventional MPD arcs are operated such that the arc speed ωl is less than that for which the departure from the square root behavior has been observed and better agreement with the actuator-disk theory is to be expected. In Fig. 6 $\omega l/a_2$ vs $IBl/\dot{m}a_2$ is plotted for the data of Refs. 1–4 and compared with the present data.

The curious property of the limiting values of $\omega l/a_2$ observed for the present experiment is that they all correspond to roughly the same value of the arc's rotational speed ωl. This lack of dependence on propellant for the limiting speed as well as for the spoke speed at which the data departs from the prediction of Eq. (4) led us to suggest that MHD effects result in the poor correlation for different propellants at high values of $IBl/\dot{m}a_2$. This follows from the fact that the magnetic Reynolds number $\mu_0\sigma Ul$ which characterizes the MHD effects depends only on the speed of the flow and the temperature of the gas and should not vary significantly for the different propellants. Using the average of the measured limiting speeds 3.4×10^4 m/sec, the magnetic Reynolds number at 12,000°K is 5.2. At this high value of the magnetic Reynolds number the MHD effects certainly cannot be neglected.

For the high values of the magnetic Reynolds number where the data for the different propellants does not correlate in Fig. 5, a better correlation can be achieved by plotting $\omega l^2\sigma\mu_0$ vs $IBl^2\sigma\mu_0/\dot{m}$. These parameters are the same as those used in Fig. 5 except that the sound speed is replaced by the speed $1/\mu_0\sigma l$, which is the plasma flow speed at which the magnetic Reynolds number is unity. Figure 7 shows that at the highest frequencies investigated the present data for all propellants approaches a common value of $\omega l^2\sigma\mu_0$ and that in general a better correlation for all the present data is observed since a large fraction of the data is at high values of the magnetic Reynolds number based on the spoke azimuthal speed. Considering the data of Refs. 1–4 and our data at corresponding values of $IBl^2\sigma\mu_0/\dot{m}$, it is questionable whether the correlation is better than that of Fig. 6. In particular the variation in frequency with molecular weight as observed by Malliaris[4] is not as well correlated by using the speed $1/\mu_0\sigma l$ in place of the sound speed a_2.

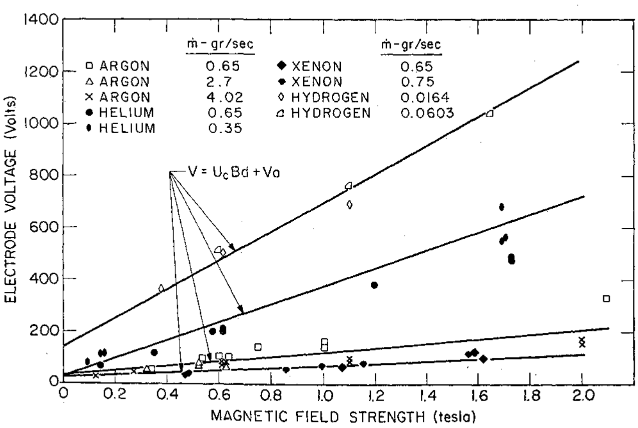

Fig. 8 Electrode voltage vs the applied axial magnetic field strength for runs with $V_eI/\dot{m}U_c^2 < 1$. Comparison is made with theory first proposed for the MPD arc by Patrick and Schneiderman.[8]

Voltage Characteristics of the MPD Arc

The most common opinion concerning the voltage characteristics of the MPD arc is expressed by the equation[8]

$$V_e = U_cBd + V_0 \tag{5}$$

where V_e is the electrode voltage, B is the applied magnetic field strength, and U_c is the critical velocity. The critical velocity is defined such that $U_c^2/2$ equals the energy to dissociate and ionize a unit mass of propellant. d has been found to be of the order of the radial electrode gap, and V_0 is interpreted as the sum of the voltage drops in the cold electrode boundary layers. For the present experiment it was found that for $V_eI \leq 2\dot{m}(\epsilon_D + \epsilon_i) \equiv \dot{m}U_c^2$, Eq. (5) correlated all the observed data, and the agreement is shown in Fig. 8. It is important to note that at these power levels the discharge was always diffuse.

For the spoke mode operation the MPD arc voltage was in general higher than the prediction of Eq. (5) and was not merely a function of the axial magnetic field strength but increased with increasing current and decreasing mass flow rate. To explain the electrode voltage of the MPD arc operating in the spoke mode, reference is made to the generalized Ohm's Law where the effect of Hall currents is neglected, which is consistent with the measurements of Ekdahl et al.[1]; i.e.,

$$E_r = j_r/\sigma + U_\theta B_z \tag{6}$$

A comparison of the present data and the prediction of Eq. (6) is shown in Fig. 9 which is a plot of the measured electrode voltage vs ωlBd. ωlBd is the back emf that would be generated by a flow rotating at the measured azimuthal spoke speed ωl. It is seen that the electrode voltage is always greater than or equal to ωlBd and becomes equal to ωlBd at the highest voltages where the voltage drops in the electrode boundary layers should be only a small percentage of the total voltage.

The voltage for the diffuse runs cannot be correlated in Fig. 9 because no measured azimuthal speed is available. However, the majority of the diffuse runs were at low-power levels such that $V_e I < \dot{m} U_c^2$ and are correlated by Fig. 8. For those runs which were diffuse and for which $V_e I > \dot{m} U_c^2$ no satisfactory correlation for all the different propellants and mass flow rates investigated has been found.

Criterion for the Existence of Spokes

The uniformity of the plasma emitted from the arc region is critical if the MPD arc is to be used as a plasma source for laboratory experiments.[18,25] For this reason it is important to be able to predict the values of I, B, and $\dot{m}$ for which the spoke mode of operation occurs for different geometries and propellants. Larson[13] has shown that the uniformity of the plasma produced is detrimentally affected by the occurrence of the spoke but that when the discharge was diffuse the flow at the nozzle exit had no time dependence and was axisymmetric. He found that in the spoke mode the ion flow was modulated at the spoke frequency, and his results suggested that there existed a strong plasma spoke closely coupled with the current spoke.

We will now proceed to suggest two different criteria each of which predict one of the observed spoke/no-spoke boundaries.

Importance of the Hall parameter

The importance of the Hall parameter $\omega_e\tau_e$ in determining the impedance characteristics and transport properties of the MPD arc was first discussed by Powers and Patrick.[6] In addition, they speculate as to its importance in making the discharge in their MPD arc axisymmetric compared to the nonaxisymmetric current distribution observed for discharges with similar electrode configurations operating without an applied axial magnetic field. It is this latter discussion that we will use now as it relates to the uniformity of the current distribution observed in the present experiment and predict the upper left-hand boundary separating the spoke and no-spoke region shown in Fig. 4.

For the diffuse discharge the azimuthal current density is related to the radial current density by the simple expression

$$j_\theta/j_r = \omega_e\tau_e = \sigma B_z/n_e e \qquad (7)$$

where n_e and e are the electron number density and the electron charge, respectively. When $\omega_e\tau_e$ is greater than one, Eq. (7) shows that the azimuthal current density exceeds the radial current density. Because of this it is the azimuthal current density that determines the dissipation in the flow j^2/σ. Since the average radial current density is fixed by the total current supplied to the MPD arc and its geometric size, Eq. (7) shows that a decrease in $\omega_e\tau_e$ will result in a decrease in the azimuthal current density. This results in a decrease in the dissipation in the flow which tends toward reducing the temperature of the flow. Based on this argument, Powers and Patrick[6] suggest a criterion for thermal stability of the discharge when $\omega_e\tau_e > 1$. This criterion is that as long as $\omega_e\tau_e$ is a decreasing function of temperature the discharge will be stable to temperature fluctuations since an increase in the temperature of the flow results in less dissipation which in turn tends to reduce the temperature. (An alternate argument which results in the same conclusion is presented in Ref. 26.)

For an ionized plasma in a constant applied magnetic field the variation of the Hall parameter is determined by the variation in τ_e the time between electron collisions. τ_e decreases as the electron number density increases and reaches a minimum when the gas becomes of the order of 90% ionized. For the MPD arc the electron number density increases as the gas temperature increases or equivalently as the arc current is increased at constant B and $\dot{m}$. Making the assumption expressed by Eq. (3) about the mass density in the arc region, the minimum value of the Hall parameter can be approximated by

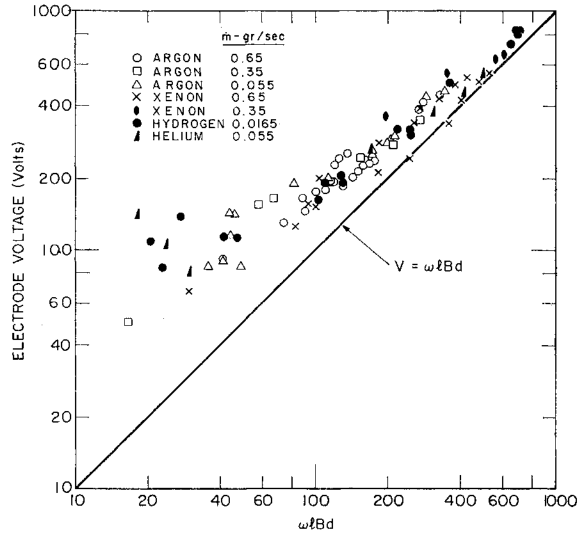

Fig. 9 Electrode voltage vs the back emf which would be generated by a conducting fluid rotating at the measured spoke speed.

$$\omega_e\tau_e = \sigma B_z m a_2 A/\dot{m}e \qquad (8)$$

where m is the molecular mass of the propellant. This value of the Hall parameter can be used in an expression for the radial current density to calculate the critical current density for a particular B and $\dot{m}$ corresponding to the minimum value of $\omega_e\tau_e$. For $\omega_e^2\tau_e^2 \gg 1$ the generalized Ohm's Law gives

$$j_r = \sigma(E_r - U_\theta B_z)/\omega_e^2\tau_e^2 \qquad (9)$$

Combining Eqs. (8) and (9) and estimating j_r as $I/2\pi ld$ and E_r as $U_c B_z$, which is a good approximation until the discharge becomes spoked,

$$IB = [(U_c - U_\theta)e^2/\sigma A](\dot{m}/ma_2)^2 \qquad (10)$$

where $A = 2\pi ld$.

Table 1 shows a comparison between the measured values of IB and those calculated by Eq. (10). In making the calculations $U_c - U_\theta$ has been assumed to be of the order of U_c and the exit gas temperature has been assumed to equal 12,000°K. It is seen that for all the propellant mass flow rates investigated except at an argon mass flow rate of 0.056 g/sec, the calculated IB is of the order of the measured limiting value. In addition the estimate is always conservative being less than that of the measured transition line. For those runs made at an argon mass flow rate of 0.056 g/sec for which the agreement is poor, the data shows that at the low current levels the discharge contains many large noncoherent fluctuations, and that at this mass flow the actual division between the nonuniform and diffuse discharge does not occur at the spoke/no-spoke boundary.

For conventional MPD arcs operating at low mass flow rates the spoke/no-spoke boundary is closer to the predicted value of Eq. (10). Larson[13] found that for his MPD arc operating at a mass flow of 25 mg/sec the discharge became spoked as the current was increased at a value of IB equal to 38 amp tesla. This compares to a value of 5.5 amp tesla calculated by Eq. (10). This inability of our MPD arc to form spokes at low mass flow rates but rather to have a discharge containing random fluctuations is not understood but is presumably caused by the difference in geometry.

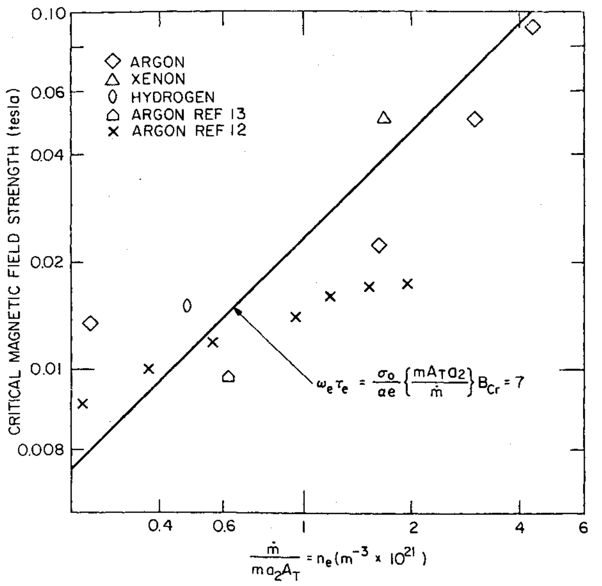

Fig. 10 Critical magnetic field below which spokes do not occur vs estimated electron number density. The line of $\omega_e\tau_e = 7$ is also included.

Critical magnetic field

Most attempts to explain the instability criterion for the MPD arc have been based on the crossed electric and magnetic field instabilities of the type which were first theoretically investigated by Simon[27] and Hoh.[28] This instability is associated with a critical magnetic field below which the discharge is stable, and an extensive experimental study has been performed to determine the value of the critical magnetic field for the MPD arc for various gases and mass flow rates.[12] In addition, a theoretical study to predict the variation of this critical magnetic field with pressure was made by Hassan.[16]

For the present experiment Fig. 4 shows that a critical magnetic field was also observed below which the discharge appeared to be more stable and rotating spokes did not occur for all values of the current investigated. Table 1 shows that the critical magnetic field is substantially lower at lower flow rates. This behavior is in qualitative agreement with the prediction made by Hassan[16] who shows that the critical magnetic field should increase with increasing pressure. A direct comparison with our data and Hassan's theory is not possible since the pressure in our MPD arc is not known. However, by using Eq. (3) to compute the density in the arc, the pressure at an estimated temperature of 12,000°K for our experiment is of the order of that considered by Hassan. A comparison of the measured critical fields with that calculated by Hassan show that the calculated values are about an order of magnitude less than the measured values.

The variation of the critical magnetic field with mass flow rate and its apparent independence of the current has suggested to us that the critical field is associated with a critical value of the Hall parameter $\omega_e\tau_e$. For a highly ionized gas, as has been shown above, the value of $\omega_e\tau_e$ can be estimated by Eq. (8). Equation (8) shows that the Hall parameter is inversely proportional to the mass flow rate and dependent on the current only as it affects the temperature of the flow. However, as discussed above, the Hall parameter is a weak function of temperature when the gas is highly ionized and therefore should be weakly dependent on the value of the current.

Figure 10 shows the variation of the critical magnetic field measured for argon, xenon, and hydrogen for the present experiments as a function of the calculated electron number density. [The electron number density has been estimated at a temperature of 12,000°K by $\dot m/a_2Am$ as was done in Eq. (8).] Figure 10 also includes the critical magnetic fields given in Refs. 12 and 13 for argon. Also plotted is the best line which correlates the data corresponding to a constant value of the Hall parameter as calculated by Eq. (8). It is seen that the data is roughly correlated by a constant value of the Hall parameter equal to 7. The actual numerical value of the Hall parameter is of little importance because of the crudeness of Eq. (8). However, considering the range of molecular weight and mass flow rates, the agreement seems more than a mere coincidence and suggests that below a critical value of the Hall parameter of order 7 the discharge is stable for all values of current.

High-current transition

At present no theoretical explanation has been found which predicts the high current transition at which the discharge changes from the spoke mode to a more uniform mode. As explained in detail in Ref. 26, the criterion suggested by the actuator-disk theory[17] did not predict the transition point.

IV. Conclusions

The experimental studies of the pulsed MPD arc permit the following conclusions:

1) Our experimentally determined voltage characteristics, spoke occurrence measurements, and spoke frequency data are consistent with that reported for conventional steady flow MPD arcs.

2) The actuator-disk theory satisfactorily correlates the spoke frequency data for all propellants and mass flow rates when the magnetic Reynolds number based on the spoke azimuthal speed is less than one.

3) When the magnetic Reynolds number based on the spoke azimuthal speed is greater than one, a better correlation of the spoke frequency can be made by plotting $\omega l^2\sigma\mu_0$ vs $IBl^2\sigma\mu_0/\dot m$, but no theoretical explanation of this is suggested.

4) The electrode voltage of the MPD arc has been found to obey the simple correlation $V_e = U_cBd + V_0$ when the total power delivered to the MPD arc satisfies the condition $V_eI < \dot mU_c^2$.

5) At higher power levels and where the MPD arc's discharge is spoked, the electrode voltage is equal to the back emf that would be generated by a flow rotating at the measured azimuthal spoke speed.

6) Our measurements indicate that the Hall parameter $\omega_e\tau_e$ is important in determining the stability of the discharge. It appears that as long as $\omega_e\tau_e$ is a strong decreasing function of temperature the discharge is diffuse and that for a highly ionized gas, as long as $\omega_e\tau_e$ is less than a value of order 7, the discharge is free of rotating spokes for all values of the current.

References

[1] Ekdahl, C., Kribel, R., and Lovberg, R., "Internal Measurements of Plasma Rotation in an MPD Arc," AIAA Paper 67-655, Colorado Springs, Colo., 1967.

[2] Larson, A. V., "Experiments on Current Rotations in an MPD Engine," *AIAA Journal*, Vol. 6, No. 6, June 1968, pp. 1001–1006.

[3] Connolly, D. J. et al., "Low Environmental Pressure MPD Arc Tests," *AIAA Journal*, Vol. 6, No. 7, July 1968, pp. 1271–1276.

[4] Malliaris, A. C., "Oscillations in an MPD Accelerator," *AIAA Journal*, Vol. 6, No. 8, Aug. 1968, pp. 1575–1577.

[5] Cann, G. L., "Annular Magnetic Hall Current Accelerator," AIAA Paper 64-670, Philadelphia, Pa., 1964.

[6] Powers, W. E., and Patrick, R. M., "Magnetic Annular Arc," *The Physics of Fluids*, Vol. 5, No. 10, Oct. 1962, pp. 1196–1206.

[7] Grossman, W., Hess, R. V., and Hassan, H. A., "Experiments with a Coaxial Hall Current Plasma Accelerator," *AIAA Journal*, Vol. 3, No. 6, June 1965, pp. 1034–1039.

[8] Patrick, R. M. and Schneiderman, A. M., "Performance Characteristics of a Magnetic Annular Arc," *AIAA Journal*, Vol. 4, No. 2, Feb. 1966, pp. 283–290.
[9] Rosciszewski, J., "Acceleration Process in the Hall Current Device," *The Physics of Fluids*, Vol. 10, No. 5, May 1967, p. 1095.
[10] Hassan, H. A., Hess, R. V., and Grossman, W., "Study of Coaxial Hall Current Accelerators at Moderate Pressures," TN D-3286, Oct. 1966, NASA.
[11] Workman, J. B., "Arc Structure in a Magnetic Annular Discharge," *AIAA Journal*, Vol. 7, No. 3, March 1969, pp. 512–519.
[12] Allario, F., Jarrett, O., and Hess, R. V., "Onset of Rotating Disturbance in the Interelectrode Region and Exhaust Jet of an MPD Arc," *AIAA Journal*, Vol. 8, No. 5, May 1970, pp. 902–907.
[13] Larson, A. V., "Measurements of Plasma Flow in an MPD Engine," AIAA Paper 69-233, Williamsburg, Va., 1969.
[14] Sidney, B. D., Allario, F., and Hess, R. V., "Onset of Rotating Disturbance in a Linear Hall Current Accelerator," *AIAA Journal*, Vol. 8, No. 6, June 1970, pp. 1117–1120.
[15] Smith, J. M., "Electrothermal Instability—An Explanation of the MPD Arc Thruster Rotating Spoke Phenomenon," AIAA Paper 69-231, Williamsburg, Va., 1969.
[16] Hassan, H. A. and Thompson, C. C., "Onset of Instabilities in Coaxial Hall Current Accelerators," AIAA Paper 69-230, Williamsburg, Va., 1969.
[17] Fay, J. A. and Cochran, R. A., "An Actuator-Disk Model for Azimuthally Non-Uniform MPD Arcs," *AIAA Journal*, Vol. 7, No. 9, Sept. 1969, pp. 1688–1692.
[18] Pugh, E. and Patrick, R., "Plasma Wind Tunnel Studies of Collision-Free Flows and Shocks," *The Physics of Fluids*, Vol. 10, No. 12, Dec. 1967, pp. 2579–2585.
[19] Schneiderman, A. M. and Patrick, R. M., "Axial Current Distribution in the Exhaust of the Magnetic Annular Arc," *AIAA Journal*, Vol. 5, No. 2, Feb. 1967, pp. 249–253.
[20] Roman, W. C. and Myers, T. W., "Experimental Investigation of an Electric Arc in Transverse Aerodynamic and Magnetic Fields," *AIAA Journal*, Vol. 5, No. 11, Nov. 1967, pp. 2011–2017.
[21] Bond, C. E., "Slanting of a Magnetically Stabilized Electric Arc in Transverse Supersonic Flow," *The Physics of Fluids*, Vol. 9, No. 4, April 1966, pp. 705–710.
[22] Winograd, Y. Y. and Klein, J. F., "Electric Arc Stabilization in Crossed Convective and Magnetic Fields," *AIAA Journal*, Vol. 7, No. 9, Sept. 1969, pp. 1699–1703.
[23] Kuethe, A. M., Harvey, R. L., and Nicolai, L. M., "Model of an Electric Arc Balanced Magnetically in a Gas Flow," AIAA Paper 67-96, New York, 1967.
[24] Cowley, M. D., "A Boundary-Layer Model for Balanced Arcs," Fluid Mechanics Lab. Publication 67-6, July 1967, Dept. of Mechanical Engineering, M.I.T., Cambridge, Mass.
[25] Pugh, E. R., Patrick, R. M., and Schneiderman, A. M., "Use of Magnetic Annular Arc Accelerator For High Enthalpy, High Stagnation Pressure Flows," *Proceedings of 8th Symposium on Engineering Aspects of MHD*, Stanford, Calif., 1967.
[26] Cochran, R. A. and Fay, J. A., "Occurrence and Behavior of Current Spokes in MPD Arcs," Fluid Mechanics Lab. Publication 70-1, Feb. 1970, Dept. of Mechanical Engineering, M.I.T., Cambridge, Mass.
[27] Simon, A., "Instability of a Partially Ionized Plasma in Crossed Electric and Magnetic Fields," *The Physics of Fluids*, Vol. 6, No. 3, March 1963, pp. 382–388.
[28] Hoh, F. C., "Instability of Penning-Type Discharge," *The Physics of Fluids*, Vol. 6, No. 8, Aug. 1963, pp. 1184–1191.

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Properties of the Rotating Spoke in an Unstable Pulsed MPD Arc

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A modified magnetoplasmadynamic (MPD) arc has been operated in a pulsed mode to permit internal probing of the arc region. The arc attains steady-state operation which strongly resembles the behavior of the more conventional continuous MPD arcs. Spatial surveys with magnetic probes and small Rogowski coil probes show the current distribution to have the form of a well defined radial spoke which rotates azimuthally. The rotation frequency is typically about 40 kc with Argon gas. Observations with double plane Langmuir probes measuring plasma flow indicate that the rotating current spoke corresponds to an actual plasma rotation. The spoke exhibits an internal structure. The plasma in the leading edge is streaming radially outward, while that in the trailing edge is counterstreaming inward towards the cathode. Most of the arc current flows in the trailing section, and the ions there seem to carry about 30% of the total arc current.

I. Introduction

THE magnetoplasmadynamic (MPD) arc has been found to be a very efficient device for converting electrical power into directed propellant kinetic energy.[1–5] Most of the early MPD research was concerned with optimization of the arc for propulsion use, which implies an emphasis on thrust, specific impulse and efficiency. Since these parameters can be determined by measurements external to the arc itself there was little motivation to investigate the precise internal character of the arc. In recent years it has become apparent that the MPD arc is indeed a complicated device requiring much more detailed information if a fundamental understanding of its operation and ultimate capabilities is to be achieved. The intent of the work discussed in this paper is to make a detailed study of the internal arc region and hopefully determine some of the more important processes which might affect the over-all arc performance. No attempt has been made to measure thrust or efficiency directly. Internal arc studies are difficult in conventional MPD arcs because of the very high power density in the current channel

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